

# Self collimation of ultrasound in a three-dimensional sonic crystal

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## Abstract

We present the experimental demonstration of self-collimation (subdiffractive propagation) of an ultrasonic beam inside a three-dimensional (3D) sonic crystal. The crystal is formed by two crossed steel cylinders structures in a woodpile-like geometry disposed in water. Measurements of the 3D field distribution show that a narrow beam which diffractively spreads in the absence of the sonic crystal is strongly collimated in propagation inside the crystal, demonstrating the 3D self-collimation effect.

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Wave beams diverge when they propagate in homogeneous materials due to diffraction. Nevertheless, a particular regime where diffraction spreading vanishes, the so-called self-collimation, was predicted a decade ago for electromagnetic waves propagating in optically periodic materials (photonic crystals) [1]. Inside a photonic crystal the dispersion relations for propagation (Bloch) modes are modified, and the envelopes of electromagnetic waves can propagate without diffraction broadening [2, 3]. The self-collimation effect has been studied not only for electromagnetic, but also for the other kind of waves. The effect analogous to the self-collimation has been recently predicted for the matter waves[4]. Also subdiffractive propagation of sonic beams inside the phononic (or sonic) crystals was predicted [5]. The vanishing of diffraction in the wave propagation along periodic crystals has been so far experimentally demonstrated for electromagnetic waves in optical [1, 6] and microwave [7] frequencies, and recently, for the ultrasonic beam propagation inside a sonic crystal [8]. However, most of the beam propagation effects, in particular the self-collimation effect on which we focus in this work, have been addressed mainly in two-dimensional (2D) systems. The three-dimensional (3D) systems are more complicated not only for the experimental study but also in the numerical level, where the FDTD calculations are extremely time consuming. From the experimental point of view, to the best of our knowledge, the 3D self-collimation has been observed only for microwaves [7] but never for the optical frequencies. Also the 3D self-collimation has never been experimentally demonstrated for the other than electromagnetic waves, i.e. the matter waves, or the sound waves. Here we demonstrate the 3D self-collimation effect in acoustics, i.e. the nondiffractive propagation of an ultrasonic beam through a 3D sonic crystal. The sonic crystal used in the experiment can be considered as formed by two squared 2D structures like that studied in [8], rotated by 90 degrees and interlaced one into another (Figure 1). Each of them are formed by 20x20 steel cylinders with a radius  $r = 0.8$  mm. The lattice constant is  $a_x = a_y = a = 5.25$  mm, where  $a_x$ ,  $a_y$  are the spatial periods along x and y direction respectively. The beam is propagated along the z direction inside the crystal. As the radius of the cylinders  $r$  is smaller than the shift between the two interlaced structures,  $a/2$  in z direction, it results in a contact-free woodpile geometry. This differs from the previously studied 3D sonic crystals, where scatterers are located forming cubic lattices with face centered cubic (fcc), body centered cubic (bcc) and simple cubic symmetries [9]. Contrary to the most common configuration in experiments using liquid-solid crystals, where in contact solid spheres are used as scatterers [10], the

main advantage of the contact-free crystal is its relatively large “transparency” reducing the energy losses in propagation and in the interfaces of the crystal.

The experimental setup consists of a source of ultrasonic wave, the above described 3D periodic structure and a needle hydrophone (to measure the acoustic field); all these components are immersed in a plexiglass tank filled with distilled water. The frequency tunable source is a piezoelectric-based commercial projector with a resonant frequency at 192 kHz, that can be tuned to a range of frequencies belonging to the second propagation band in of our crystal (200 to 260 KHz) where the experiment is performed. The needle hydrophone is an Onda Corp. HNR-0500 with a calibrated operating band between 0.25 and 10 MHz, that can be used for stable relative measurements in the frequency range of interest. The acoustic signals are generated and captured using a PXI National Instruments system with synchronized signal generator and oscilloscope cards with oversampling frequency capability. To position the hydrophone, three motorized axis are governed by the acquisition system, mapping the acoustical beam. The excited signal is a tone burst of several cycles of the studied frequency; the pulse is long enough to assume the CW propagation inside the crystal but sufficiently short to discard unwanted reflections coming from the tank walls in the captured signal. The measured pressure levels are low, assuring the linear regime. In order to reduce the noise the temporal averaging of several pulses (up to 20) is performed and a Butterworth eighth order band-pass filter is applied. The 3D sonic crystal, as explained above, can be considered as two interlaced 2D crystals studied in [5, 8]. In the 2D case the nondiffractive propagation was predicted for two different frequencies in the first and second bands respectively, and experimentally verified for the second band (225 kHz). Here, in the present letter, we study experimentally the propagation through the 3D crystal in the second band by varying the frequency around the above mentioned 2D self-collimation frequency (225 kHz).

Figure 2 shows the effect of the crystal on the propagation of the beam. Fig. 2(a) shows the sound intensity distributions in the transversal planes just after transducer, i.e. at a distance of 3 mm from the transducer plane. Fig. 2(b) shows the distributions in a free (without crystal) propagation over the distance of 115 mm, respectively. The propagated beam is slightly anisotropic, i.e. is slightly broader in the horizontal direction because the adapting layer of the emitting transducer has a certain curvature (astigmatism) in that plane, acting as a cylindrical diverging lens. Fig. 2(c) shows the amplitude profile of the sound

beam at the rear face of the crystal, measured at the same 115 mm distance as in Fig. 2(b). The diameter of the central part of the beam remains nearly of the same order than the input (just slightly broadened), clearly indicating the effect of self-collimation. Besides the central self-collimated beam, the side-lobes appear which correspond to the diverging wave vectors. The side-lobes are related with the excitation of the additional Bloch modes (in addition to the basic subdiffractive one), and require a separate study. We note that these side-lobes disappear after the larger distances behind the sonic crystal (not shown). Fig. 3 depicts the variation of the beam width (measured at half amplitude) versus frequency. A minimum is reached for the self-collimation frequency of the 2D case [8]. The width is normalized here to the width of the beam after propagating the same distance without crystal, and computed subtracting the noise level amplitude with no interpolation between the spatially sampled points. The depicted width is the average of the different measurements of the width for the different vertical cuts of the transversal plane.

The continuous curve in Fig. 3 is the theoretical fit. In paraxial treatment, the beam broadening after the propagation in a homogeneous material with diffraction coefficient  $d$  is given by  $\Delta x^2(z) = \Delta x_0^2 + (4 d z)^2 / \Delta x_0^2$ , where  $\Delta x_0$  is the initial width and  $z$  the propagation distance. This classical formula for the propagation of Gaussian beams can be extended to the case of inhomogeneous media, taking into account the particular dependence of the diffraction coefficient with the frequency. In our case, we can assume to a first order approximation that the diffraction coefficient depends linearly with frequency, and that changes the sign (it cancels) at the self-collimation point. The other coefficients are selected as those with a better fit to the experimental data. We note that our 3D crystal is in fact the enlace of two orthogonally oriented 2D structures, therefore the spatial modulation  $m$  of the acoustic parameters (bulk modulus and density) is the additive function of corresponding 2D crystals:  $m(x, y, z) = m_1(x, z) + m_2(y, z)$ , where the both functions depend separately on  $x$  and  $y$ . Under paraxial approximation, where the field propagation is described by the Schrödinger equation, the acoustic field (pressure field,  $p$ ) factorize in this case of additive potentials:  $p(x, y, z) = p_1(x, z) \times p_2(y, z)$ . The 3D propagation problem then simplifies into two independent 2D propagation problems studied before in [5] and [8]. Strictly speaking the propagation in the studied by us 3D sonic crystal is not paraxial, since the sound waves diffract at relatively large angles in every slice. However, having in mind that the self-collimation phenomenon is usually analogous in paraxial and nonparaxial cases (see e.g.[11]

for comparison of the self-collimation in these cases) the similarity of the measured sound profiles in 3D case with product of 2D profiles [8], as well as the good coincidence between the self-collimation frequencies in 3D and 2D cases is plausible. In conclusion, a non-contact woodpile sonic crystal, consisting of two interlaced 2D structures of periodic arrays of steel cylinders in water, has been fabricated. Experiments of ultrasonic beam propagation through such crystal have demonstrated the self-collimation propagation in a 3D sonic crystal. Two additional remarks should be done about the advantages of the used crystal: on one hand, the crystal differs from the most common close-packed solid spheres crystals in the sense that, in the present crystal the filling fraction can be arbitrarily modified allowing to design the crystal without the restriction existent in close-packed spheres, where the filling fraction is fixed at  $f_{max} = 0.7405$ . In our case, the filling factor is  $f = 147$ , i.e. twice than in the 2D case. On the other hand, the scatterers in the type of crystals used by us are not in contact which could enable to introduce a source inside the crystal [12] without any additional perturbation. This should enable the characterization of the directional propagation inside the 3D crystal, which is not possible in the close-packed spheres structures, where a defect should be introduce in the structures to locate the emitter inside.

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# Figure Captions

Fig.1- (a) Unit cell scheme and (b) photograph of the crystal used in the experimental setup.

Fig. 2- Transverse profile of the ultrasonic beam measured at (a) 3 mm from the trasducer, (b) at 115 mm from the trasducer in free propagation and (c) at the crystal output located at 115 mm from the trasducer, after propagating through the crystal.

Fig. 3. Beam width versus frequency, dots represent the experimental points with the corresponding dispersion bars, and the analytical fit (see text below) is depicted by the continuous line.